RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, JUNE 2022

SECOND YEAR [BATCH 2020-23]

Date : 23/06/2022 **PHYSICS (HONOURS)**

Time : 11 am – 1 pm **Paper : IX [CC9]** Full Marks : 50

Answer **any five** questions: [5×10]

1. a) Deduce and explain D'Alembert.s principle.

   b) Find out using this principle the acceleration of a body sliding down a frictionless inclined plane.

   c) Explain briefly the advantages of the Lagrangian formalism over the Newtonian formulation. [4+4+2]

2. a) A pendulum is suspended from the ceiling of a lift descending with an acceleration f. Discuss the motion of the system using Lagrange's equation. What happens if f=g ?

   b) Two Lagrangians differing the total time derivative of any function f(q,t) give the same equation of motion. Use this to demonstrate Galilean invariance of the laws of mechanics.

   c) What is a cyclic coordinate? Explain its significance with an example. [4+3+3]

3. a) The Lagrangian L= 1/2 mv.v - q(V- v.A) , where V and A are the scalar and the vector potentials respectively. Find out the Hamiltonian. Does it represent the energy of the system? Explain.

   b) Deduce the number of degrees of freedom of a rigid body. Explain the precession of a torque free symmetric rigid body with the body set of axes taken as the principal axes. [4+6]

4. a) Two masses m1 and m2 are connected by a weightless spring and the system is executing motion in the vertical direction. Discuss the motion for small displacement.

   b) If F and H are constants prove that F is explicitly independent of time. Deduce the necessary formula. Discuss the motion for small displacement using Lagrange's equation. [4+6]

5. a) Why Michelson-Morley experiment was performed? If the two arms of a Michelson interferometer have lengths l_1 and l_2, show that the fringe shift when the interferometer is rotated by 90^o with respect to the velocity v through the ether is $\Delta N = \frac{l_1+l_2}{\lambda}\frac{v^2}{c^2}$, where λ is the wavelength of the light.

   b) If the optical path length $l_1 + l_2 = 22$ m and light source was yellow line of sodium $\lambda = 590$ nm. Determine the fringe shift using the orbital velocity of Earth around the Sun is 30 km/s.

   c) How the 'Ether drag' hypothesis can explain the null result of Michelson-Morley experiment? Explain briefly why we have to give up the 'Ether drag' hypothesis. [(1+5)+1+(1+2)]

6. a) Use the Lorentz transformation to determine the time dilation effect.

   b) A young man voyages to the nearest star, α Centauri, 4.3 light-years away. He travels in a spaceship at a velocity of $\frac{c}{5}$. When he returns to earth, how much younger is he than his twin brother who stayed home.

   c) The space and time coordinates of two events as measured in a frame S are as follows:

   Event 1: $x_1 = x_0, t_1 = \frac{x_0}{c}, y_1 = z_1 = 0$ Event 2: $x_2 = 2x_0, t_2 = \frac{x_0}{2c}, y_2 = z_2 = 0$

(i) There exist a frame in which these events occur at the same time. Find the velocity of this frame with respect to S.

(ii) What is the value of t' at which both events occur in the new frame?

d) If an event A precedes an event B at the same point in one frame of reference, will A precede B in all other inertial frames? Will they occur at the same point in any other inertial frame? Will the time interval between the events be the same in any other inertial frame? Explain.

[3+2+(1+1)+(1+1+1)]

7. a) What is a four vector? How can you construct invariant quantities (under Lorentz transformation) using four vectors?

b) Derive the relativistic equations of Doppler effect using space-time four vector $\underline{X} = (\vec{x}, ict)$ and propagation four vector $\underline{K} = (\vec{k}, i\omega/c)$.

(i) Show that the relativistic formula reduces to the classical one of $v/c \ll 1$.

(ii) The wavelength of a spectral line measured to be λ on Earth found to increase by 50% on a far distant galaxy. What is the speed of the galaxy relative to Earth? [(1+1)+(5+2+1)]

8. a) In Newtonian mechanics, the kinetic energy of a mass m moving with velocity $\vec{v}$ is $K = \frac{mv^2}{2} = \frac{p^2}{2m}$, where $\vec{p} = m\vec{v}$. Hence, the change in kinetic energy due to a small change in momentum is $dK = \frac{\vec{p}.d\vec{p}}{m} = \vec{v}.d\vec{p}$. Show that the relation $dK = \vec{v}.d\vec{p}$ also holds in relativistic mechanics.

[Hint: $E^2 = p^2c^2 + m_0^2c^4$]

b) A particle of rest mass m and speed v collides and stick to a stationary particle of mass M. What is the final speed of the composite particle?

c) In S frame the components of forces are given by

$F_x = \frac{d}{dt}(mu_x), F_y = \frac{d}{dt}(mu_y)$ and $F_z = \frac{d}{dt}(mu_z)$

whereas the corresponding quantities in S'-frame are

$F'_x = \frac{d}{dt'}(m'u'_x), F'_y = \frac{d}{dt'}(m'u'_y)$ and $F'_z = \frac{d}{dt'}(m'u'_z)$

Using Lorentz and velocity transformation express forces in one frame in terms of forces in another frame. [Hint: $E = \gamma(E' + vp'_x)$ and $\frac{1}{\sqrt{1-u^2/c^2}} = \frac{\gamma\left(1+\frac{u'_x v}{c^2}\right)}{\sqrt{1-u'^2/c^2}}$, where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$] [2+3+5]

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